Int. J. Heat Mass Transfer. Vol. 21, pp. 481–488. Pergamon Press 1978. Printed in Great Britain

MEASURED VARIATION OF THERMAL BOUNDARY-LAYER THICKNESS WITH PRANDTL NUMBER FOR LAMINAR NATURAL CONVECTION FROM A VERTICAL UNIFORM-HEAT-FLUX SURFACE

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(*Received* 27 *August* 1976 *and in revised form* 7 *April* 1977)

Abstract—This investigation presents systematic measurements of the variation of the thermal boundary-layer thickness with Prandtl number for a Prandtl number range of from 0.703 to 8940 for laminar natural convection from a vertical, uniform-heat-flux surface. Experimental measurements of the thermal boundary-layer thickness, using a differential thermocouple probe, were made in five fluids: air, water, ethylene glycol, tri-ethylene glycol, and SF-97-1000 silicone fluid. In addition, a Schlieren system was used to visualize the thermal boundary-layer region in each fluid. Very good agreement was found between the experimental results and the thermal boundary-layer thickness predicted by both the well-known similarity solution and the recent correlation of Churchill and Ozoe. The largest disagreement was found for the highest Prandtl number fluid. For the high Prandtl number silicone fluid the temperature field was also determined for several values of heat flux. While good agreement with theoretical results was found near the surface, somewhat higher temperatures were measured near the edge of the thermal boundary layer. The larger difference at high Prandtl number is attributed to the non-boundary-layer nature of the velocity field and the associated disturbance of the flow by the presence of the test section boundaries.

NOMENCLATURE

c_p, fluid specific heat;
g, acceleration due to gravity;
G^*, flux Grashof parameter, $5(Gr_x^*/5)^{1/5}$;
Gr_x^*, local flux Grashof number, $g\beta x^4 q''/k\nu^2$;
Gr_x, local Grashof number, $g\beta x^3(t_0 - t_\infty)/\nu^2$;
k, fluid thermal conductivity;
N, defined by, $t_0 - t_\infty = Nx^{0.2}$, for uniform flux surface;
q'', heat flux from vertical surface;
r, electrical resistivity of Inconel 600;
t, temperature;
x, coordinate along vertical surface;
y, coordinate normal to vertical surface.

Greek symbols

β, volumetric coefficient of fluid thermal expansion;
δ, boundary-layer thickness;
Δt, temperature difference, $t - t_\infty$;
η, dimensionless similarity variable, $(y/x)(Gr_x/4)^{1/4}$;
μ, absolute viscosity of fluid;
ν, fluid kinematic viscosity, μ/ρ;
ρ, fluid density;
σ, fluid Prandtl number, $\mu C_p/k$;
ϕ, temperature excess ratio, $(t - t_\infty)/(t_0 - t_\infty)$.

Subscripts

co, denotes Churchill and Ozoe;
exp, value determined experimentally;
f, film temperature, $(t_0 + t_\infty)/2$;
0, condition at surface, $y = 0$;
ss, similarity solution;
t, the edge of the thermal boundary layer;
∞, condition in ambient fluid.

INTRODUCTION

THERE have been many previous studies of steady laminar natural convection from a vertical uniform-heat-flux surface. One of the principle analytical investigations was a similarity analysis of the boundary layer equations by Sparrow and Gregg [1]. The governing equations, in non-dimensional form, contain a single parameter, the fluid Prandtl number. Numerical solutions were presented for Prandtl numbers: 0.1, 1, 10 and 100. Definitive analyses for extreme Prandtl numbers occurred much later than the corresponding ones for an isothermal surface. By considering an inner and outer layer, Hieber and Gebhart [2] and Hieber [3] obtained three terms in a series solution valid for the limiting situations of large and small Prandtl number, respectively, using the method of matched asymptotic expansions. The leading terms, for heat transfer, are tabulated by Gebhart [4] for extreme Prandtl numbers for both the isothermal and uniform flux surface boundary condition.

Perhaps the most detailed and extensive experimental measurements of natural convection adjacent to a vertical uniform-heat-flux surface have been performed in air and water. Some of the earliest theoretical and experimental studies for liquids were done by Saunders [5] in water and mercury using an electrically heated surface. The measured temperature profiles and heat transfer agreed quite well with his corresponding approximate solutions for these liquids. Later measurements of local heat transfer in air by Dotson [6] agree well with the results of Sparrow and Gregg [1] for $10^5 < Gr_x < 10^{11}$. Additional measurements by Goldstein [7] in air and water also support theoretical results.

Many of the investigations using fluids of higher Prandtl number than air and water, and a uniform flux surface, were concerned with transient and non-Newtonian effects. Soehngen [8] measured the temperature profile in hydrocarbon polymers (σ = 50 000–200 000) as part of a study of transient and steady-state natural convection. He concluded that his measured steady state temperature profile confirmed the trends predicted by similarity solution calculations for an isothermal surface, but a careful comparison of experiment and theory was not presented. Rajan and Picot [9] experimentally determined the Nusselt number dependence on Prandtl number for transient and steady-state laminar natural convection from a vertical uniform-heat-flux surface over a range of Prandtl number from 6 to 10^6. They measured the heat transfer from a uniform-flux surface in four fluids (water, glycol, glycerol, and corn syrup) and correlated their results using a modified version of the equation given by von Karman–Pohlhausen (Sparrow and Gregg [1]). Recent measurements in mercury reported by White *et al.* [10] indicate that natural convection heat-transfer rates in liquid metals are higher than those predicted by current analytical methods.

In a very recent study, Emery *et al.* [11] measured the effect of inclination and non-Newtonian behavior on heat transfer to fluids with Prandtl numbers ranging from 7 to 1000. For the high Prandtl number fluids, they found that the presence of test section boundaries gave rise to secondary flows which slightly alter the temperature field from that calculated using similarity solutions.

Churchill and Ozoe [12] have presented rather general heat-transfer correlations for laminar natural convection from both isothermal and uniform flux vertical surfaces. Their results agree with similarity solution predictions, for moderate values of Prandtl number, and asymptotic solutions for limiting large and small Prandtl number. In the course of their calculations they also determined an expression for the thermal boundary-layer thickness (for a uniform flux surface) which is valid for all Prandtl numbers.

A knowledge of the Prandtl number dependence of the thickness of the momentum and thermal boundary layers is of considerable practical and fundamental importance. For example, it may be desirable to avoid interference with the thermal boundary layer locally to prevent unwanted heating of an object adjacent to a heated surface. Additionally, an estimate of the boundary-layer thickness is often necessary to determine the applicability of the boundary-layer approximations. For moderate Prandtl numbers, the thermal boundary-layer thickness is inversely proportional to the square root of the Prandtl number ($\delta_t \propto \sigma^{-1/2}$). This is a consequence of viscous forces in the vertical direction being of the same order as the convection of momentum and the motion-causing buoyancy force (see Gebhart [4]). For vanishingly small Prandtl number this relation becomes exact. However, at large Prandtl number the thermal region thickness varies inversely as the one-fourth power of the Prandtl number. Knowledge of the Prandtl number variation of the thermal and momentum boundary-layer thicknesses is of fundamental importance in separating the flow into inner and outer regions and employing the technique of matched asymptotic expansions.

In connection with high Prandtl number fluids, it is interesting to note a number of observations of Hieber and Gebhart [2]. They point out that the boundary-layer approximation for the inner region is valid when $G^* = O(\sigma^{3/10})$. They also indicate that the outer region does not have a significant effect upon the inner, since the rate of change of vertical momentum in the outer layer is of smaller order than the viscous shear force. They conclude that their stability analysis is valid even though it corresponds to a flow regime where the outer layer of the flow region is not fully developed.

Although asymptotic theories are heavily relied upon to predict flow and transport at extreme Prandtl number, there remain some unanswered and important questions. At low Prandtl number, White *et al.* [10] point out a lack of agreement between experiment and theory. At large Prandtl number, experiments have dealt mainly with other effects, and careful comparisons of velocity and temperature fields and transport have not been made. A verification of the Prandtl number dependence of the thermal boundary-layer thickness is an examination of an important part of the basis of asymptotic theories. The effect of Prandtl number on the thermal boundary-layer thickness predicted by analysis has not previously been systematically verified experimentally over an extensive range of Prandtl number. Such a verification is the purpose of the present experimental investigation. No previous investigations have presented a detailed comparison of measured and calculated velocity and temperature fields for Prandtl numbers greater than 600. The experimental apparatus and procedure are described in the next section.

EXPERIMENTAL APPARATUS

A uniform-heat-flux surface was constructed of a thin (0.127 mm thick) electrically heated foil made of Inconel 600 stretched between two metal knife edge clamps. The foil was 6.9-cm wide and 27.4-cm long. The thickness of the foil is very uniform and the material is homogeneous to high tolerance. The re-

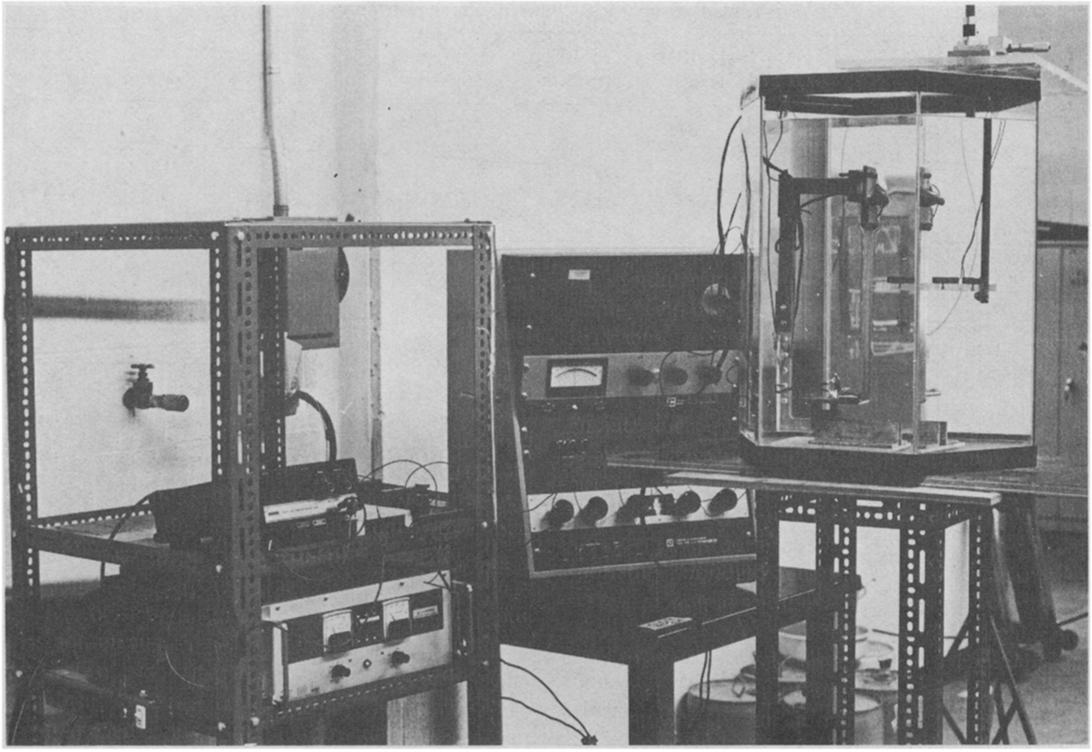

FIG. 1. Photograph of experimental apparatus.

sistivity of the foil is a very weak function of temperature, $r = r_r[1+1.70\times10^{-4}(t-t_r)]$, which insures that the heat dissipation will be essentially uniform along the surface even though the temperature will vary along its length $(t_0 - t_\infty = Nx^{-0.2})$. The largest temperature variation along the foil occurred in air. From one end to the other, the maximum temperature difference was 77°C. This produced only a 1% change in the resistance along the entire length of the foil. Convection occurs on both sides of the foil so that half of the electrical energy dissipated in the foil is convected away on each side.

The temperature difference between a point in the boundary layer and the ambient fluid was measured using a differential thermocouple probe. Small (0.025 mm) diameter thermocouple wires were stretched between small diameter glass tubes held in a fixture. This design minimized the disturbance of the flow in the vicinity of the thermocouple bead. Also, since the boundary layer is nominally two-dimensional, the leads from the thermocouple were positioned horizontally, along an approximate isotherm, which minimized conduction errors. The second bead of the differential thermocouple was located at the opposite end of the probe in the ambient fluid outside the thermal boundary layer. The ambient temperature was measured using a precision thermometer. A K-4 Leeds and Northrup potentiometer was used to measure the voltage from the differential thermocouple probe. At the top of the test section a micrometer was used to determine the distance of the probe from the foil.

A Schlieren optical system was also used to observe the thermal boundary layer in each fluid. A fine thread reference grid was placed in the optical path just outside the test section so that the distance from the probe to the leading edge of the foil could be measured from the photograph of the boundary layer.

A conventional electrical circuit was used to measure the power dissipated from the foil. The voltage across the foil and across a Leeds and Northrup precision 0.1 Ω (±0.00004 Ω) resistor in series with the foil were measured with a digital voltmeter (to within ±0.0001 V). The voltage across the standard resistor was used to calculate the current in the circuit. The power dissipated in the foil was thus determined to within ±0.01%. Measurements were made in five fluids: air, water, ethylene glycol, tri-ethylene glycol and SF-97-1000 silicone fluid. The properties of these fluids at the film temperature, t_f, are shown in Table 1. The tank and associated experimental apparatus are shown in Fig. 1. Additional details of the experimental apparatus can be found in Carey [13], part I.

NEW MEASUREMENTS AND RESULTS

The measurement of the thermal boundary-layer thickness in each fluid was made at a value of heat flux chosen to be within the regime of validity of the theoretical solutions. Higher heat flux would tend to increase the temperature difference at the edge of the boundary layer (taken to be about $\phi = 0.02$, see Table 2 for exact values), and to decrease the physical size of the boundary layer. Since there are limitations on the capability of the instruments to make these measurements (±0.006 K for the thermocouple probe and ±0.025 mm for the boundary-layer thickness

Table 1. Properties of fluids at film temperature

Fluid	t_f (K)	ρ (kg/m^3)	$\beta \times 10^5$ (1/K)	$\nu \times 10^6$ (m^2/s)	μ (Ns/m^2)	c_p (J/kgK)	k (W/mK)	σ
Air	325.2	1.13	3.071	18.14	2.055×10^{-5}	1.004	0.028	0.703
Water	293.0	998	20.34	1.015	1.012×10^{-3}	4.176	0.598	7.06
Ethylene glycol	299.7	1101	63.58	13.27	1.023×10^{-2}	2.364	0.285	137
Tri-ethylene glycol	300.2	1136	68.56	33.44	2.933×10^{-3}	2.113	0.233	344
Silicone oil	300.2	972	92.52	968.0	9.413×10^{-1}	1.506	0.159	8940

Table 2. Tabulated experimental parameters for the data shown in Fig. 3

Fluid	σ	t_∞ (K)	$q'' \times 10^2$ (W/cm^2)	N (K/cm$^{0.2}$)	x (cm)	y (mm)	$t_0 - t_\infty$ (K)	Gr_x	ϕ_{exp}	η_{exp}	η_{theory}
Air	0.703	290.2	3.47	39.47	17.9	12.3	70.2	3.76×10^7	0.0213	3.820	3.790
Water	7.06	289.7	16.83	3.62	18.7	4.6	6.51	8.56×10^7	0.0188	1.658	1.580
Ethylene glycol	137	294.4	12.25	6.32	19.1	4.9	11.5	2.20×10^6	0.0137	0.702	0.709
Tri-ethylene glycol	344	293.3	9.37	6.96	19.1	5.1	12.3	5.11×10^5	0.0268	0.504	0.497
Silicone oil	8940	294.9	2.08	5.09	18.3	15.1	9.11	5.44×10^2	0.0191	0.280	0.227

measurements), to keep the percentage error as small as possible for both, a compromise was made on the value of heat flux used. In addition, many of the fluid properties are strongly temperature dependent, specifically: ρ, μ, k and c_p. The similarity solution entails a number of assumptions including that of constant fluid properties. Therefore, large values of heat flux were avoided to reduce property variation in the flow field and insure that the similarity solution is an accurate representation of the physical phenomena. Another assumption used in the similarity solution entails the second part of the Boussinesq approximation: $\beta\Delta t \ll 1$. Again, the temperature difference across the boundary layer was kept low to ensure that this condition was not violated. All of the approximations used in the boundary-layer similarity solution are carefully considered and shown to be valid in Carey [13], part I.

Care was taken when conducting the experiment to ensure that a steady-state condition was achieved without allowing the fluid in the tank to become thermally stratified. As the Prandtl number of the fluid increased, it required more time to reach steady state. The problem of thermal stratification in the tank was minimized by insulating the tank and by thoroughly stirring the fluid for approximately one hour before running the experiment. As a check, the highly sensitive Schlieren system was employed to detect thermal gradients in the tank.

For convection in air, the heat loss due to radiation can be a significant portion of the total heat dissipated (about 30% of the total for the case considered here). Since this effect is not included in the similarity analysis, we have corrected our measurements in air using the perturbation analysis results of Audunson and Gebhart [14]. With both modes of heat transfer present the convective heat flux varies weakly with x. The details of this analysis are discussed in Carey [13], part I. It can be concluded that the convective heat flux closely approximates a constant value (within 5%, for the experiments described here) over the length of the foil even with radiation present. Additionally, the very close agreement of our radiation corrected measurements in air are a vindication of the theoretical results of Audunson and Gebhart [14].

Measuring the thermal boundary-layer thickness in the high Prandtl number silicone fluid presented additional concerns. An estimate of the relative boundary-layer thickness indicated that if the thermal boundary layer is 1.3-cm thick, then the velocity boundary layer is of the order of 130-cm thick. Since the test tank is 51-cm across at its widest point (it is a hexagonal prism 61-cm tall with 25-cm sides), the velocity boundary layer was disturbed by the tank walls. Boundary-layer flow requires δ/x to be small. Since x is only about 15 cm, the velocity field comprises a non-boundary-layer flow regime. The temperature field is, however, a boundary-layer configuration, i.e. $\delta_t/x \ll 1$.

Previous analytical work by Hieber and Gebhart [2] and others consider free convection at high Prandtl numbers to consist of two distinct regions. The inner region is the thermal boundary layer where the temperature difference is brought to zero. The outer region consists of the distance beyond the inner region required for the velocity profile to go to zero. They further imply that the thermal boundary layer and the outer velocity field are only weakly coupled (mathematically only through the matching conditions at the interface of the two regions). This suggests, therefore, that the interference of the velocity boundary layer by the tank walls may have little effect on the thermal boundary layer, and consequently the heat transfer so long as the thermal boundary layer is sufficiently thin (δ_t/x small). As previously mentioned, Hieber and Gebhart [2] concluded that the boundary-

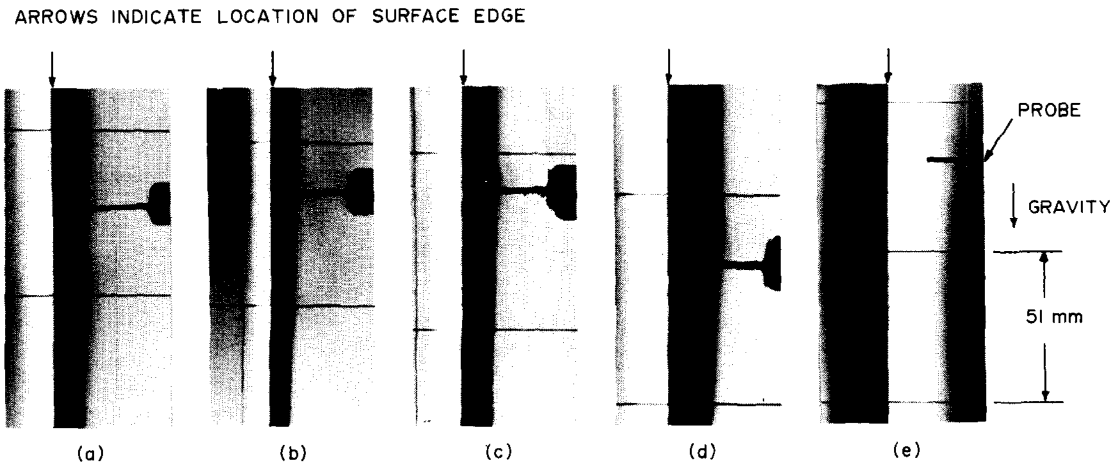

FIG. 2. Schlieren photographs of the thermal boundary layers: (a) Air ($\sigma_f = 0.703$); (b) Water ($\sigma_f = 7.06$); (c) Ethylene glycol ($\sigma_f = 137$); (d) Triethylene glycol ($\sigma_f = 344$); (e) SF-97 silicone fluid ($\sigma_f = 8940$).

layer approximation for the inner region of the flow, is valid for $G^* = O(\sigma^{3/10})$. Our experimental conditions were chosen to insure that the thermal boundary layer satisfied this condition ($G^* = 25$ and $\sigma^{3/10} = 15$).

Another consideration, particularly important regarding the silicone fluid, was that the viscosity is a very strong function of temperature. The kinematic viscosity of the fluid is 0.001 m^2/s at 298 K. At 333 K the viscosity has dropped to 0.0005 m^2/s. The viscosity change was minimized by keeping the temperature difference across the boundary layer low. Once the differential thermocouple probe was positioned, and the flow reached steady state, the thermocouple voltage was read and a Schlieren photograph was taken.

For air, water, ethylene glycol and triethylene glycol the temperature was measured at a single location near the edge of the thermal boundary layer. For the silicone fluid, the entire temperature profile was measured at a single downstream location for several values of heat flux. The high precision of the instrumentation used, as previously discussed, resulted in an accuracy of better than 1% for the experimentally determined values of η_t and ϕ, listed in Table 2 along with associated experimental parameters. Also shown in Table 2 are the values of η_t, from our numerical calculations, corresponding to the value of ϕ determined experimentally. The last two columns of Table 2 are a direct comparison of the experimental measurement of thermal boundary-layer thickness and the values predicted by the numerical integration of the similarity solution equations. For the highest Prandtl number calculations, the slopes of the velocity and temperature profiles were taken from Hieber and Gebhart [2].

As a check on the numerical solution, the value of η_t for $\phi = 0.01$ was also calculated using the relation given by Churchill and Ozoe [12]. Their expression, modified to give η_t directly is:

$$\eta_t = \frac{3.410\left[1+\left(\dfrac{\sigma}{7.22}\right)^{9/10}\right]^{2/9}}{[-\phi'(0)]^{1/5}\sigma^{2/5}}.$$

A comparison of the numerical solution results with those predicted by the Churchill–Ozoe relation is shown in Table 3. It can be seen that they are in agreement to at least within the 2% accuracy claimed by Churchill and Ozoe [12].

Table 3. Comparison of the similarity solution results and those predicted by the Churchill–Ozoe correlation for $\phi = 0.01$

Fluid	σ	η_{ss}	η_{co}
Air	0.703	4.410	4.510
Water	7.06	1.740	1.750
Ethylene glycol	137	0.728	0.718
Tri-ethylene glycol	344	0.575	0.564
SF-97 silicone fluid	8940	0.248	0.247

In Fig. 2, Schlieren photographs of the thermal boundary layers are shown collectively for air (a), water (b), ethylene glycol (c), tri-ethylene glycol (d), and SF-97-1000 silicone fluid (e). The photographs were taken immediately after the thermocouple measurements were recorded. The vertical line seen in some of the photographs was aligned with a vertical plumb line. In all the photographs the horizontal lines were 5.1 cm apart and the probe is located between the line that was 15.2 cm from the leading edge and the one that was 20.3 cm from the leading edge. For each fluid the distance of the probe from the lower line was determined by measuring the distance on a blow-up of the Schlieren photograph (to twice the physical size). It can be seen that the structure of the thermal boundary layer is essentially the same in each fluid. The weak growth of the boundary layer with x can also be seen. The thickness of the thermal boundary layer in the vicinity of the probe is small compared to the distance from the leading edge.

Figure 3 shows the variation of η_t with Prandtl number. Calculated curves of η_t corresponding to values of ϕ equal to 0.01, 0.02, and 0.03 are shown. The experimental values of η_t at each value of Prandtl number are also shown on the plot. In addition, the

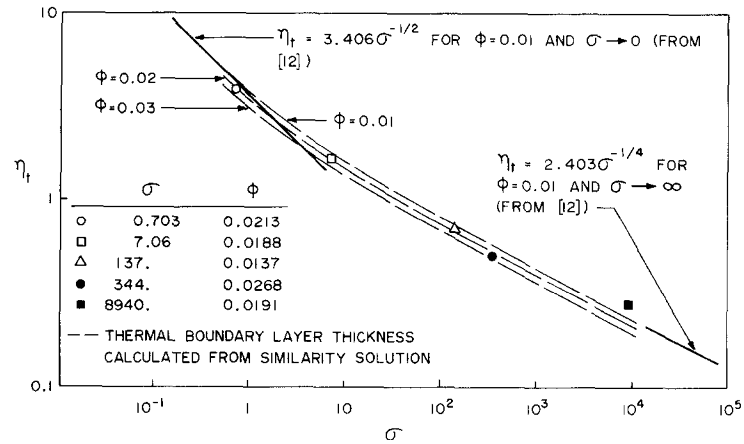

FIG. 3. Variation of the thermal boundary-layer thickness with Prandtl number.

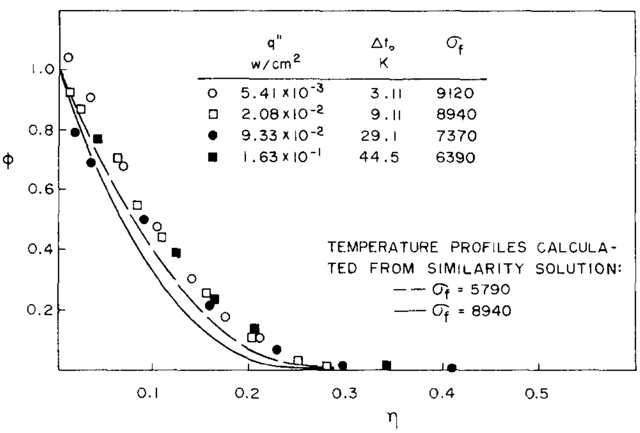

FIG. 4. Measured and calculated temperature distribution for the silicone fluid.

asymptotic Prandtl number dependence for extreme Prandtl number is shown for $\phi = 0.01$. These relations are from the Churchill and Ozoe [12] correlation and incorporate the relations for the Prandtl number variation of $-\phi'(0)$, for extreme values of Prandtl number, from Gebhart [4]. It can be seen from Fig. 3 (and Table 2) that the experimental results agree very closely with the predicted values, with the exception of the point in the silicone fluid which is about 20% high. Because of this, more extensive data was taken at high Prandtl numbers to examine the nature of the boundary layer in more detail. The temperature field in the boundary layer was measured for four values of heat flux. The results are shown in Fig. 4. Because the viscosity is such a strong function of temperature, changing the heat flux changed the film Prandtl number of the fluid. This resulted in a range of film Prandtl numbers from 6.4×10^3 to 9.1×10^3. Analytical solutions were determined for $\sigma_f = 8940$ and 5790 and are plotted in Fig. 4. The larger value corresponds exactly to the film Prandtl number of one set of experimental conditions. The lower value is plotted to indicate how much the calculated temperature field changes with Prandtl number.

It can be seen from Fig. 4 that for all four values of heat flux, the experimental values of ϕ are consistently higher than the values predicted by the similarity solution. Note that measurements were made over a very wide range of heat flux. The agreement is good close to the foil, but for $\eta > 0.05$ the experimental values of ϕ differ from the calculated values by as much as 50%. Three potential sources of error have been considered. First, the difference could be caused by the strong variation of kinematic viscosity of the silicone fluid with temperature. The other thermal properties of the silicone fluid (c_p and k) do not vary strongly with temperature and therefore should not greatly affect the flow. Calculated Prandtl number variation is shown on Fig. 4. Close examination of Fig. 4 shows that the error between the experimental data points and the theoretical curves is greater for the data points that were obtained at the lower values of $t_0 - t_\infty$. This implies that the variation of the transport properties (particularly the kinematic viscosity) is probably not the primary cause of the error. For the experimental conditions (i.e. calculated shear rates encountered) here, the deviation of the fluid from non-Newtonian behavior is negligible. Another possibility is that the difference could be caused by the interference of the thick velocity boundary-layer with the wall of the tank. This effect is believed to be a possible source of error and is discussed further in the next section.

CONCLUSIONS

For all Prandtl numbers considered, values of the thermal boundary-layer thickness, η_t, calculated from the similarity solution agree to within 2% of values calculated from the correlation of Churchill and Ozoe [12]. Except for measurements in the high Prandtl number silicone fluid, our experimentally determined values of η_t agree extremely well with the theoretical predictions. The good agreement found for measurements in air, after correcting for radiation losses, serves also to further verify the results of Audunson and Gebhart [14]. They found that for radiation effects as small as those present in this study, the radiation and convective transport can be considered to be independent and superimposed.

The Prandtl number variation of η_t from order-of-magnitude analysis for moderate values of Prandtl number, and asymptotic analyses for extreme values of Prandtl number are also compared with numerical calculations in Fig. 3. The inverse square root variation is seen to be a rather rough estimate at moderate Prandtl number compared to the calculated variation. For the small Prandtl number limit, the inverse square root variation agrees well with the limited calculations shown for $\sigma < 1$. At large Prandtl number, the calculated variation is seen to be in close agreement with the inverse one-fourth variation beyond $\sigma = 100$.

The experimentally determined temperature profile in the high Prandtl number silicone fluid deviates somewhat from that predicted by the similarity solution. Although it may be a contributing factor, the variation of viscosity with temperature does not appear to be the primary source of error because the error does not increase with increasing temperature difference. A conjecture from a previous analytical study on the stability of high Prandtl number free convection flows by Hieber and Gebhart [2] suggests that the thermal boundary layer and the outer velocity field are not strongly coupled. If the two layers are sufficiently decoupled, it may be expected that the theory would be an accurate representation of the temperature profile (and heat transfer) despite the non-boundary-layer nature of the velocity field.

Calculations show that the velocity field in our experiments in the high Prandtl number silicone fluid is thick compared to the downstream extent considered. In addition to the non-boundary-layer velocity field, there is an associated effect. The velocity field is calculated to extend beyond the test section boundaries. It is interesting to note that the trends indicated by our measured temperature profile at high Prandtl number are similar to those found by Emery *et al.* [11]. They found, for high Prandtl number fluids, that the test section boundaries produced a recirculating flow which affected the velocity field and the outer region of the thermal boundary layer. Their measured temperature profile agreed well with that predicted by the similarity solution except near the edge of the thermal region where the experimental data is slightly higher. Our data, shown in Fig. 4, has the same characteristics. It agrees well with the similarity solution near the surface, but the data is higher near the edge of the thermal boundary layer. It seems likely, therefore, that our measurements could also be influenced by secondary motions in the ambient fluid resulting from flow interaction with the test section enclosure.

The only circumstance where we do not have excellent agreement between our measurements and theoretical calculations is for the highest Prandtl number considered. Furthermore, for the other fluids used, both the velocity and temperature fields are thin enough not to interfere with test section boundaries and to satisfy boundary-layer requirements. We have considered and ruled-out a number of other possible explanations for the disagreement found in the silicone fluid. The remaining candidate is the effect of interference of the thick non-boundary layer velocity field with test section boundaries. The results of this investigation imply that the assumption of insignificant coupling between the thermal boundary layer and the outer velocity field is questionable when there is interference with test section boundaries.

In view of this, and the close similarity of our measurements to those of Emery *et al.* [11], we tentatively conclude that enclosure effects are responsible for the theoretical underprediction of the temperature in the boundary layer for the high Prandtl number fluid. Furthermore, the disagreement in the outer region is somewhat greater in the present study than that reported by Emery *et al.* [11]. This suggests, perhaps, that the effect becomes more pronounced with increasing Prandtl number. Some previous studies in high Prandtl number fluids have apparently overlooked this effect and their results are thus called into question.

Acknowledgements—The authors would like to acknowledge support for this research under National Science Foundation Grant ENG 76-16936 and by the Institutional Funds Committee, SUNY/Buffalo, and Harrison Radiator Division of General Motors for donating two of the fluids used in this investigation. In addition, we acknowledge Mr. Harold Wagner for his technical assistance with the experimental apparatus. We further acknowledge Prof. B. Gebhart for his many contributions to this work.

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MESURE DE LA VARIATION DE L'EPAISSEUR DE COUCHE LIMITE THERMIQUE EN FONCTION DU NOMBRE DE PRANDTL POUR LA CONVECTION NATURELLE LAMINAIRE SUR UNE SURFACE VERTICALE ET UN FLUX THERMIQUE UNIFORME

Résumé—Des mesures systématiques de la variation de l'épaisseur de couche limite thermique ont été conduites pour des nombres de Prandtl variant de 0,703 à 8940, en convection naturelle laminaire sur une surface verticale chauffée à flux constant. Des mesures de l'épaisseur de couche limite thermique à l'aide d'une sonde différentielle à thermocouple, ont été faites pour cinq fluides: air, eau, éthylène glycol, triétylène glycol et silicone SF-97-1000. De plus un système Schlieren a été utilisé pour visualiser la région de couche limite dans chaque fluide. Un très bon accord a été obtenu entre les résultats et les calculs tirés de la solution classique de similarité et de la récente formule de Churchill et Ozoe. Le plus grand désaccord est trouvé pour les nombres de Prandtl les plus élevés. Pour le silicone à grand nombre de Prandtl, le champ de température est aussi déterminé pour plusieurs valeurs du flux thermique. Tandis qu'un bon accord avec les résultats théoriques est trouvé près de la surface, des températures plus élevées sont mesurées près du bord de la couche limite. La différence au grand nombre de Prandtl est attribuée à une nature du champ des vitesses qui s'écarte de celle d'une couche limite et à la perturbation associée de l'écoulement par la présence des frontières de la section de mesure.

MESSUNG DER VERÄNDERUNG DER THERMISCHEN GRENZSCHICHTDICKE MIT DER PRANDTL-ZAHL BEI LAMINARER, NATÜRLICHER KONVEKTION AN EINER VERTIKALEN HEIZFLÄCHE MIT GLEICHFÖRMIGER WÄRMESTROMDICHTE

Zusammenfassung—Im Rahmen dieser Untersuchung wurden systematische Messungen der Veränderung der thermischen Grenzschichtdicke mit der Prandtl-Zahl bei laminarer, natürlicher Konvektion an einer vertikalen Heizfläche mit gleichförmiger Wärmestromdichte durchgeführt. Die Prandtl-Zahlen wurden von 0,703 bis 8940 variiert. Die thermische Grenzschichtdicke wurde mit Hilfe einer Differential-Thermoelement-Sonde für fünf verschiedene Fluide gemessen, nämlich fur Luft, Wasser, Aethylenglykol, Triäthylenglykol und SF-97-1000 Silikonöl. Außerdem wurde die Grenzschichtregion mit Hilfe eines Schlierenverfahrens sichtbar gemacht. Die experimentell ermittelte Grenzschichtdicke stimmt sowohl mit der nach der Aehnlichkeitstheorie wie mit der nach der kürzlich veröffentlichten Beziehung von Churchill und Ozoe errechneten sehr gut überein. Die stärksten Abweichungen treten bei Fluiden hoher Prandtl-Zahlen auf. Für das hohe Prandtl-Zahlen aufweisende Silikonöl wurde auch das Temperaturfeld für verschiedene Wärmestromdichten bestimmt. Während sich eine gute Übereinstimmung mit den theoretischen Ergebnissen in der Nähe der Oberfläche ergibt, wurden an den Rändern der thermischen Grenzschicht etwas höhere Temperaturen gemessen. Die bei höheren Prandtl-Zahlen auftretenden Abweichungen werden auf den Nichtgrenzschichtcharakter der Strömung und auf von den Rändern der Versuchsstrecke ausgehende Störgrößen zurückgefürt.

ВЛИЯНИЕ ЗНАЧЕНИЯ ЧИСЛА ПРАНДТЛЯ НА ТОЛЩИНУ ТЕПЛОВОГО ПОГРАНИЧНОГО СЛОЯ ПРИ ЛАМИНАРНОЙ ЕСТЕСТВЕННОЙ КОНВЕКЦИИ ОТ РАВНОМЕРНО НАГРЕТОЙ ВЕРТИКАЛЬНОЙ ПОВЕРХНОСТИ

Аннотация — В статье представлены результаты исследования влияния числа Прандтля на толщину теплового пограничного слоя в диапазоне значений числа Прандтля от 0,703 до 8940 при ламинарной естественной конвекции от равномерно нагретой вертикальной поверхности. Замеры толщины теплового пограничного слоя производились дифференциальным термоэлементом в пяти средах: воздухе, воде, этиленгликоле, триэтиленгликоле и в силиконе SF-97-1000. Помимо этого, область теплового пограничного слоя визуализировалась в каждой из сред теневым методом. Наблюдалось хорошее соответствие между экспериментальными данными и результатами расчетов толщины теплового пограничного слоя, полученными как из автомодельных решений, так и с помощью недавно предложенного соотношения Черчилля и Озое. Наибольшее расхождение отмечено у жидкости с самым большим значением числа Прандтля. Для силиконовой жидкости с большим значением числа Прандтля определялось температурное поле при нескольких значениях плотности теплового потока. В то время как на поверхности отмечалось хорошее соответствие с результатами расчетов, у края теплового пограничного слоя замерены более высокие значения температуры. Большее расхождение результатов при высоких значениях числа Прандтля объясняется характером поля скоростей, отличным от свойственного пограничному слою, и сопутствующим возмущением потока, обусловленным наличием границ экспериментального участка.